where λ is the free space wavelength. Eq. (2) shows that an increase in the radius of the guide will decrease the characteristic impedance. It is desirable to minimize the change in the impedances at the junction. Therefore, a larger diameter is preferred for matching the circular guide to the rectangular waveguide.

From condition 1) above it it seen that the necessary and sufficient condition for propagation of the dominant TE_{11} mode while suppressing all higher order modes is

$$\frac{\lambda l}{3.41} < a \leq \frac{\lambda h}{2.61}, \tag{3}$$

where λl is the wavelength at the low end of the band and λh is the wavelength at the high end of the band.

From condition 2) above it is seen that the optimum transformer design occurs at the maximum permissible radius. Thus

$$\frac{\lambda l}{3.41} < a = \frac{\lambda h}{2.61}. \tag{4}$$

The author would like to acknowledge many helpful discussions with S. Lehr, and R. Mohr. He is also indebted to L. Bertan, who supervised the project, and J. Ebert for their many helpful suggestions.

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A Microwave Power Divider*

Recent literature has described the theoretical performance of unmatched power dividers[1,2]

A proposed multilaterally matched power divider for any number n of equal or unequal

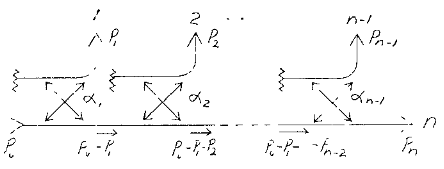

FIG 1—Directional coupler power divider.

outputs, is shown in Fig. 1, where

P_i = input power to the divider
P_k = output power from the kth output port
α_k = power coupling coefficient of the kth coupler =

$$\frac{P_k}{P_i - \sum_{q=1}^{k-1} P_q}.$$

The synthesis of the divider to provide n outputs of prescribed values with a given input is straightforward. The various α's are solved for from the relation

$$\alpha_k = \frac{P_k}{P_i - \sum_{q=1}^{k-1} P_q} \qquad 1 \leq k < n \tag{1}$$

since

$$P_i = \sum_{q=1}^{n} P_q \tag{2}$$

from energy considerations, the choice of all P_q, and hence all α_q from $q=1$ to $q=n-1$, quite determines P_n.

The isolation α_{lm} between output ports l and m is

$$\alpha_{lm} = \alpha_l \alpha_m \alpha_D \tag{3}$$

where all α's are in power ratios and α_D is the directivity of the coupler nearest the input. This is a minimum isolation, since resistive and coupling losses to intervening couplers are neglected.

The divider proposed is 100 per cent efficient; it is matched looking into any port; the isolation between output ports is infinite (assuming perfect directivity); further, there is no theoretical limit to the number of outputs or relative amplitude of outputs that may be obtained consistent with (2).

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* Received by the PGMTT, July 31, 1961.

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On the Efficiency of the Excitation of Surface Waves by Distributed Coupling*

INTRODUCTION

The excitation of surface waves on reactive surfaces is accompanied by loss of power which is radiated directly from the region of the feed. Since a surface wave supported by a surface wave line is a (nonhomogeneous) plane wave, it cannot be excited as the only field of a current distribution of finite size and amplitude.

The excitation efficiency is defined as that fraction of the total power transmitted through the exciting aperture, which is contained in the surface wave field. Excitation efficiencies approaching theoretically computed values have been achieved in practice by using horizontal or annular slots, but these apertures usually presented to the primary line highly reflecting loads, and consequent introduction of matching structures made the system rather frequency sensitive.[1]

Several methods have been described for the measurement of the excitation efficiency; most of these depend on some kind of measurement on the reactive surface.[2,3]

The method for the excitation of surface waves which is described in this paper is essentially an application of the theory and technique of directional couplers.[4] In our case, however, it is sought to achieve complete power transfer from the primary line onto the reactive surface waveline. The theoretical treatment will therefore be based on Miller's light coupling theory.[5] The reader is referred to Miller's paper for a complete and systematic analysis of a system of coupled transmission lines. Here we shall summarize, with the aid of Fig. 1, the most important results of this analysis.

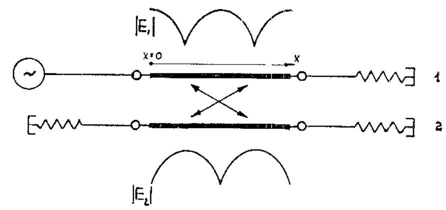

Fig. 1—A system of coupled transmission lines.

1) When two homogeneous transmission lines are coupled along the axis of propagation, power transfer between the lines takes place cyclically.

2) If, and only if, both lines have identical propagation constants, *i.e.*, $\gamma_1 = \gamma_2 (\gamma_n = \alpha_n + j\beta_n)$, complete power transfer is possible, and the minimum length of the coupling aperture necessary is given by $2cx_{\min} = \pi$ where c is the coupling coefficient in nepers per unit length.

Complete power transfer is also possible when $\gamma_1 - \gamma_2 = \alpha_1 - \alpha_2 > 0$, but in this case $x_{\min}$ will be different from the value given above, and generally, in the presence of losses, the term *complete power transfer* will mean only that values of x exist for which no power is present in line 1.

3) When $\gamma_1 \neq \gamma_2$, and in particular when $\beta_1 \neq \beta_2$, only partial power transfer will take place. The maximum possible wave amplitude in line 2 is, in this case, a function of $(\beta_1 - \beta_2)/c$ and is defined as the discrimination function of the coupled system. In this case, again, the point of maximum possible power transfer will differ from the value of x given for $\gamma_1 - \gamma_2 = 0$.

* Received by the PGMTT, August 3, 1961. This note is a sequel to the author's report on "Improvement of the Excitation Efficiency of Surface Waves," M.S. thesis, Technion, Israel Inst. Tech., Haifa, Israel, January, 1960.

[1] Because of the numerous contributions to the subject dealt with in this note the reader is referred to two survey papers which contain exhaustive bibliographies.
a) F. J. Zucker, "The guiding and radiation of surface waves," *Proc. Symp. on Modern Advances in Microwave Techniques*, Polytechnic Institute of Brooklyn, N. Y.; 1954.
b) A. F. Harvey, "Periodic and guiding structures at microwave frequencies," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol MTT-8, pp. 30–61; January, 1960.

[2] G. Goubau, "On the excitation of surface waves," PROC. IRE, vol. 40, pp. 865–868; June, 1952.

[3] R. H. DuHamel and J. W. Duncan, "Launching efficiency of wires and slots for a dielectric rod waveguide," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 277–284; July, 1958.

[4] R. J. Hanratty, "An end-fire X-band flush antenna based on the branch-waveguide directional coupler," private communication.

[5] S. E. Miller, "Coupled wave theory and waveguide applications," *Bell Sys. Tech. J.*, vol. 33, pp. 661–719; May, 1954.

The main difference between this method of surface wave excitation and methods which employ apertures having zero length lies in the absence of a single discontinuity in the primary line. As a result, the bandwidth of such a system is determined almost exclusively by the dependence of $\gamma_1 - \gamma_2$ and of c on frequency.

Theory

Fig. 2 shows a length of rectangular waveguide which has a series of closely spaced slots cut in one of its broad walls.

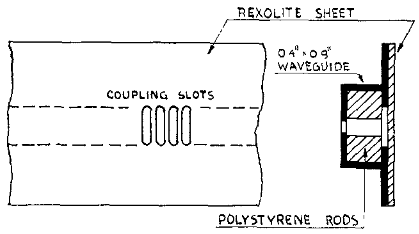

Fig. 2—Section of a waveguide-to-reactive surface directional coupler.

This wall extends on the outside to form the metal base of a dielectric clad reactive surface. We now assume that when a propagating wave is excited in the waveguide, a fraction of the power carried by it is coupled into a surface wave supported by the reactive surface and part of it is radiated into surrounding space. We further assume that the waveguide and reactive surface possess the characteristics of a system of homogeneous coupled transmission lines. With each of these lines, we associate a propagating constant of the lowest-order propagating mode, *viz.*,

$$\gamma_1 = \alpha_1 + j\beta_1 \tag{1a}$$

$$\gamma_2 = \alpha_2 + j\beta_2 \tag{1b}$$

where the subscripts 1 and 2 refer to the waveguide and reactive surface, respectively.

The propagating wave amplitudes in the coupled system satisfy the set of equations

$$\frac{dE_1}{dx} = -(\gamma_1 + jc)E_1 + jcE_2 \tag{2a}$$

$$\frac{dE_2}{dx} = -(\gamma_2 + jc)E_2 + jcE_1 \tag{2b}$$

where

E_1 = complex wave amplitude in the waveguide
E_2 = complex surface wave amplitude
c = coupling coefficient between waveguide and surface wave field.

Only propagating waves in the positive x direction are assumed to exist, and coupling action starts at $x=0$ with the initial conditions

$$E_1(0) = 1; \qquad E_2(0) = 0.$$

We now consider the orthogonality relations valid for the surface wave field and radiation field and use these relations in order to postulate the existence of two separately defined coupling coëfficients: one for the waveguide-to-surface wave field and another for the waveguide-to-radiation field. The radiation field is excited by the slots, and the contribution from a particular slot is proportional to $E_1(x)$ at that slot. Hence, in an otherwise lossless system, this coupling coefficient is represented by the attenuation coefficient $\alpha = \alpha_1$ of the primary line. We finally assume that $\beta_1 = \beta_2 = \beta$, and we shall show later that the validity of this assumption can be verified experimentally. Eqs. (2a) and (2b) thus take the form:

$$\frac{dE_1}{dx} = -[\alpha + j(\beta + c)]E_1 + jcE_2 \tag{3a}$$

$$\frac{dE_2}{dx} = -j(\beta + c)E_2 + jcE_1 \tag{3b}$$

and their solution for $E_1(0)=1$; $E_2(0)=0$ is

$$|E_1| = \frac{e^{-cx\xi}}{\sqrt{1-\xi^2}} \cdot \left| \cos\left(cx\sqrt{1-\xi^2} + \operatorname{arctg} \frac{\xi}{\sqrt{1-\xi^2}} \right) \right| \tag{4a}$$

$$|E_2| = \frac{e^{-cx\xi}}{\sqrt{1-\xi}} \cdot \left| \sin cx\sqrt{1-\xi^2} \right| \tag{4b}$$

where $\xi = a/2c$.

From 4(a), we find that zero field amplitudes will be detected in the primary line at points x_n given by

$$cx_n\sqrt{1-\xi^2} + \operatorname{arctg} \frac{\xi}{\sqrt{1-\xi^2}} = (2n-1)\frac{\pi}{2} \quad (n = 1, 2 \cdots) \tag{5}$$

and, in particular, position coordinates of the first two minima x_1 and x_2 satisfy

$$cx_1\sqrt{1-\xi^2} + \operatorname{arctg} \frac{\xi}{\sqrt{1-\xi^2}} = \frac{\pi}{2} \tag{6a}$$

$$cx_2\sqrt{1-\xi^2} + \operatorname{arctg} \frac{\xi}{\sqrt{1-\xi^2}} = \frac{3\pi}{2} \tag{6b}$$

from which we obtain

$$c = \frac{\dfrac{\pi}{\alpha_1(\rho - 1)}}{\cos \dfrac{\pi}{2} \dfrac{\rho - 3}{\rho - 1}} \tag{7}$$

and

$$\xi = \sin \frac{\pi}{2} \frac{o - 3}{\rho - 1} \tag{8}$$

where $\rho = x_2/x_1$. We now substitute from (7) and (8) into (4b) and put $x = x_1$, which is the point of maximum surface wave amplitude. Thus

$$|E_2|_{\max} = \exp\left\{ -\frac{\pi}{\rho - 1} \cdot \tan \frac{\pi}{2} \frac{\rho - 3}{\rho - 1} \right\} \cdot \frac{\sin \dfrac{\pi}{\rho - 1}}{\cos \dfrac{\pi}{2} \cdot \dfrac{\rho - 3}{\rho - 1}} = \exp\left\{ -\frac{\pi}{\rho - 1} \cdot \tan \frac{\pi}{2} \cdot \frac{\rho - 3}{\rho - 1} \right\}. \tag{9}$$

(The fraction equals unity identically.)

For $E_1(0) = 1$, the surface wave excitation efficiency η is defined by

$$\eta = |E_2|^2_{\max} \tag{10}$$

and thus

$$\eta = \exp\left\{ -\frac{2\pi}{\rho - 1} \cdot \tan \frac{\pi}{2} \cdot \frac{\rho - 3}{\rho - 1} \right\}. \tag{11}$$

Eq. (11) is an expression of the excitation efficiency in terms of the distances to the points of maximum power transfer. This can be generalized by stating that *in the case of surface wave excitation by distributed coupling it is possible, in principle, to determine the excitation efficiency by measuring the distribution of the absolute value of the wave amplitude in the primary line only.*

Fig. 3 shows the plot of η vs ρ for the range $3.01 \leq \rho \leq 4$. For values of ρ in the range $3 \leq \rho \leq 3.01$, the approximation

$$\eta \cong 1 - \frac{\pi}{4}(\rho - 3) \tag{12}$$

can be used and the plot of this function is shown in Fig. 4.

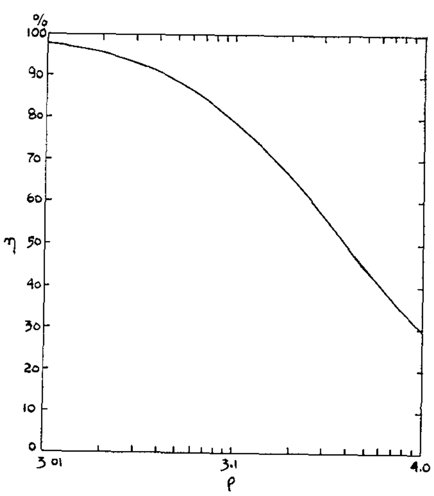

Fig. 3—Plot of excitation efficiency vs displacement of field minima in primary line for $3.01 \leq \rho \leq 4$.

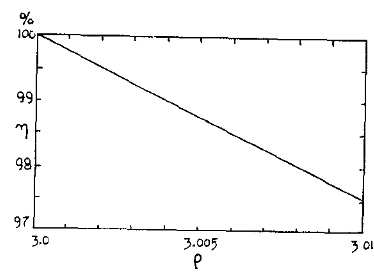

Fig. 4—Plot of excitation efficiency vs displacement o field minima in primary line for $3 \leq \rho \leq 3.01$.

Experiment

It was pointed out in the Introduction that complete power transfer, in the restricted sense, is possible only if $\beta_1 = \beta_2$. Thus, the detection of zero field amplitude in the primary line can serve, in principle, as an indication of the equality of phase coefficients of the coupled lines.

The initial design of the system called for the insertion of dielectric slabs into the waveguide, and the resulting value of β_1 was determined with the aid of a method described by the author elsewhere.[6] The thickness of the dielectric sheet to give the necessary value of $\beta_2=\beta_1$ was determined from the transverse resonance condition.[1a] Because of inaccuracies involved in this procedure, the design did not result in the desired situation in which $\beta_2=\beta_1$ at the design frequency. At this stage, use has been made of the fact that the group velocities in the coupled lines have different values, *i.e.*, $\partial\omega/\partial\beta_1 \neq \partial\omega/\partial\beta_2$, and by searching in the neighborhood of the design frequency, a particular frequency has been found for which $\beta_1=\beta_2$.

Copper-clad teflon, and later Rexolite, have been used in the construction of the reactive surface. The coupling aperture consisted of a row of 150 slots cut in the wide wall of a 0.4×0.9-in ID rectangular waveguide (see Fig. 2). The measurement of the field amplitude in the waveguide was performed by cutting in the opposite wide wall a longitudinal slot such as used in a slotted section and mounting the waveguide in a hp809B Universal Probe Carriage from which the original slotted section has been removed. The whole setup has been placed on a surface covered with microwave absorbing material.

In a few experiments conducted up to now, excitation efficiencies of between 92 and 95 per cent have been determined.

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Higher-Order Mode Resonances in Strip-Line Y-Junction Circulators*

Because of its small physical size, the Y-junction circulator has been the recipient of a great amount of investigation. Various theorists and experimentalists have devoted time to understanding the behavior of this device, both on the far and near side of ferrimagnetic resonance.[1–4]

In our laboratory, we have succeeded in developing units operating at the near side of resonance from 1.0–8.0 kMc. These units have instantaneous bandwidths providing 20 db of isolation in excess of 20 per cent; some units have bandwidths as high as 40 per cent. One of the limitations on bandwidth that we have discovered are resonances that occur in the insertion loss and isolation characteristics of the device at the HF end of the band. In narrow-band designs, these resonances might never be noticed. Because of their presence, it is necessary to limit the bandwidth specification to exclude them from the operating region. If resonance could be eliminated, we believe bandwidths of 50–60 per cent could be achieved.

Fig. 1 shows some experimental points relating the frequency at which the resonance occurs to the diameter of the ferrimagnetic material used in the design. Various materials were used, and they are labeled with their manufacturer's designation.

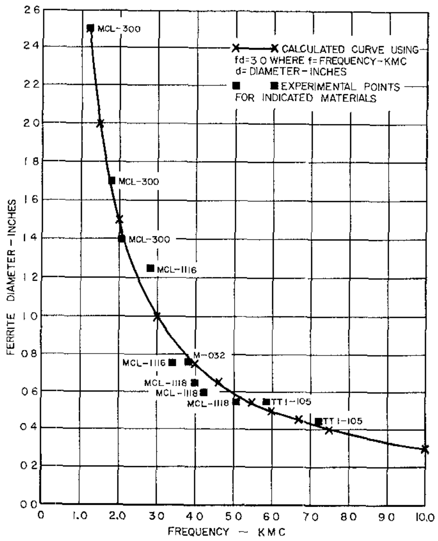

Fig. 1—Diameter at which higher-order mode resonance occurs for strip-line Y-junction circulators on the near side of resonance.

The frequency of these resonances depends upon the value of magnetic field. They occur at lower frequencies for smaller field values. The values of field used in the data presented were sufficient to saturate the disks.

A theoretical curve is plotted using the relationship

$$fd=3.0$$

where

f = frequency in kMc
d = diameter of material in inches.

This type of expression is typical of propagation in cylindrical waveguide where the mode cutoff frequency is related to the guide diameter and the proper-order Bessel function. It is felt, therefore, that these resonances can be explained by higher-order mode propagation in a direction parallel to the applied magnetic field, even though the dominant mode in the strip line is propagating perpendicular to this direction.

It is hoped that these resonances can be moved to higher frequencies without effecting the basic nonreciprocal scattering of the ferrimagnetic disks. Experiments to determine the feasibility of removing these resonances are now underway and, if successful, will be reported.

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* Received by the PGMTT, August 21, 1961.

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[4] J. Clark and J. Brown, "Miniaturized, temperature stable, coaxial Y-junction circulators," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES (Correspondence), vol. MTT-9, pp. 267–269; May, 1961.

A Proposed Design to Enhance Microwave-Power-Limiter Characteristics*

A design is proposed for a device which would enhance the properties of presently available microwave power limiters, thereby making their use as crystal protectors in duplexing units of microwave systems more desirable. This design is a combination of a power-sensitive, nonlinear element with a traveling-wave ring resonator.

Several nonlinear elements, such as subsidiary resonance ferrite limiters,[1] and DeGrasse type of ferrimagnetic limiters,[2] and diode parametric limiters[3] have been devised which exhibit an attenuation vs power-level curve such as is depicted in the lower curve of Fig. 1. The nonlinear properties of ferroelectric materials indicate that these materials might also be used to produce limiting action.

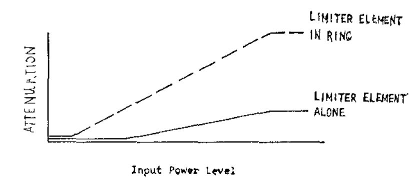

Fig. 1.

However, the referenced limiters are not completely usable as crystal protectors because either their threshold power levels are too high, or their maximum attenuations, slopes of attenuation vs power curve, or their power-handling capabilities are inadequate. Combining any one of the nonlinear elements with the traveling-wave ring resonator would improve upon all of these shortcomings, provided that the low-level insertion loss of the element is sufficiently small.

* Received by the PGMTT, August 21, 1961.

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